

# Connectedness In Bitopological Spaces

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The concept of connectedness a fundamental notion in general topology investigates the structural property of a space being in one piece or not separable In traditional topology a topological space is deemed connected if it cannot be expressed as the union of two non empty disjoint open sets However the study of bitopological spaces endowed with two topologies enriches the understanding of connectedness by introducing a more intricate interplay between the two structures This article delves into the fascinating world of connectedness in bitopological spaces exploring various definitions properties and their implications

## Bitopological Spaces A Brief Overview

A bitopological space is a set equipped with two topologies Formally a bitopological space is a triple  $(X, \tau_1, \tau_2)$  where  $X$  is a set and  $\tau_1$  and  $\tau_2$  are topologies on  $X$  The presence of two topologies allows for a richer analysis of topological properties including connectedness

## Types of Connectedness in Bitopological Spaces

In bitopological spaces the concept of connectedness takes on several forms each capturing a different aspect of the interplay between the two topologies The most common types include

- 1 **Pairwise Connectedness** A bitopological space  $(X, \tau_1, \tau_2)$  is said to be pairwise connected if there exist no nonempty  $\tau_1$ open and  $\tau_2$ open sets that are disjoint This definition directly extends the traditional notion of connectedness to the bitopological setting
- 2  **$ij$ Connectedness** For  $i, j \in \{1, 2\}$  with  $i \neq j$  a bitopological space  $(X, \tau_1, \tau_2)$  is  $ij$  connected if there exist no nonempty  $\tau_i$ open and  $\tau_j$ closed sets that are disjoint This type of connectedness explores the interaction between open sets in one topology and closed sets in the other
- 3 **Weakly Connectedness** A bitopological space  $(X, \tau_1, \tau_2)$  is weakly connected if there exist no nonempty  $\tau_1$ open and  $\tau_2$ open sets that are disjoint and whose union equals the whole space This definition focuses on the inability to decompose the space into completely separated open sets from both topologies
- 4  **$\tau_1$ Connectedness and  $\tau_2$ Connectedness** A bitopological space  $(X, \tau_1, \tau_2)$  is  $\tau_1$ connected if it is connected with respect to the topology  $\tau_1$  and similarly  $\tau_2$ connected if it is connected with respect to the topology  $\tau_2$  These notions correspond to the traditional concept of connectedness applied to each topology individually

**Properties and Relationships** The different types of connectedness in bitopological spaces exhibit interesting relationships and properties. Pairwise connectedness implies  $i$ - $j$  connectedness for all  $i, j$  in  $1, 2$  with  $i \neq j$ . This follows directly from the definitions as disjoint  $\tau_i$ -open and  $\tau_j$ -closed sets are also disjoint  $\tau_i$ -open and  $\tau_j$ -open sets. Pairwise connectedness does not imply weak connectedness. Consider a bitopological space with two topologies, one being the discrete topology and the other being the indiscrete topology. This space is pairwise connected but not weakly connected.  $i$ - $j$  connectedness for both  $i, j$  in  $1, 2$  with  $i \neq j$  implies weak connectedness. This holds because if the space is not weakly connected, it can be decomposed into two disjoint open sets violating the  $i$ - $j$  connectedness condition.  $\tau_1$ -connectedness and  $\tau_2$ -connectedness do not imply any of the other types of connectedness. This is because each topology is considered individually, ignoring the interaction between them.

**Examples and Applications**

**Product Spaces** Given two topological spaces  $X_1, \tau_1$  and  $X_2, \tau_2$ , their product space  $X_1 \times X_2, \tau_1 \times \tau_2$  is pairwise connected if and only if both  $X_1, \tau_1$  and  $X_2, \tau_2$  are connected.

**Function Spaces** The space of continuous functions from a topological space  $X, \tau$  to a topological space  $Y, \sigma$ , denoted by  $C(X, Y)$ , can be equipped with different topologies such as the compact-open topology and the pointwise convergence topology. The connectedness properties of these function spaces depend on the specific topologies chosen.

**Digital Topology** Bitopological spaces find applications in digital image processing where the two topologies are often chosen to represent the connectivity of objects in digital images. For instance, one topology might represent the 4-connectedness of pixels while the other represents the 8-connectedness.

**Conclusion** The study of connectedness in bitopological spaces provides a richer understanding of topological properties by considering the interplay of two different topological structures. The various types of connectedness, including pairwise connectedness,  $i$ - $j$  connectedness, weak connectedness, and connectedness with respect to individual topologies, offer a nuanced framework for analyzing the connectedness of bitopological spaces. This research area has diverse applications, from topological investigations to digital image processing, highlighting the significance of extending classical topological concepts to the bitopological setting.

Some Concepts in Bitopological Spaces  
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 Functional Analysis in

Asymmetric Normed Spaces Quasi-components in Bitopological Spaces Neutrosophic Soft Bitopological Spaces Neutrosophic Sets and Systems, Vol. 38, 2020 Scientia Magna, Vol. 5, No. 3, 2009 Bitopological Function Spaces Applications of a Conditional Preopen Sets in Bitopological Spaces Neutrosophic Soft Cubic Set in Topological Spaces Proceedings of the Indian Science Congress On Generalizations of the Notion of Continuity of Multifunctions in Bitopological Spaces Mathematical Reviews Generalizations of Lindelof Properties in Bitopological Spaces Kyungpook Mathematical Journal Topology ON I-OPEN SETS AND I-CONTINUOUS FUNCTIONS IN IDEAL BITOPOLOGICAL SPACES National Academy Science Letters Journal of the Indian Institute of Science Edwin Victor Selander Badri Dvalishvili Suman Das Stefan Cobzas I. L. Reilly Ahmed B. AL-Nafee Florentin Smarandache Zhang Wenpeng Yong-un Kim Hardi Nasralddin R. Anitha Cruz Indian Science Congress Association Andrzej Rychlewicz Zabidin Salleh Ákos Császár M. CALDAS National Academy of Sciences, India Indian Institute of Science, Bangalore Some Concepts in Bitopological Spaces Bitopological Spaces: Theory, Relations with Generalized Algebraic Structures and Applications Pairwise Neutrosophic-b-Open Set in Neutrosophic Bitopological Spaces Functional Analysis in Asymmetric Normed Spaces Quasi-components in Bitopological Spaces Neutrosophic Soft Bitopological Spaces Neutrosophic Sets and Systems, Vol. 38, 2020 Scientia Magna, Vol. 5, No. 3, 2009 Bitopological Function Spaces Applications of a Conditional Preopen Sets in Bitopological Spaces Neutrosophic Soft Cubic Set in Topological Spaces Proceedings of the Indian Science Congress On Generalizations of the Notion of Continuity of Multifunctions in Bitopological Spaces Mathematical Reviews Generalizations of Lindelof Properties in Bitopological Spaces Kyungpook Mathematical Journal Topology ON I-OPEN SETS AND I-CONTINUOUS FUNCTIONS IN IDEAL BITOPOLOGICAL SPACES National Academy Science Letters Journal of the Indian Institute of Science *Edwin Victor Selander Badri Dvalishvili Suman Das Stefan Cobzas I. L. Reilly Ahmed B. AL-Nafee Florentin Smarandache Zhang Wenpeng Yong-un Kim Hardi Nasralddin R. Anitha Cruz Indian Science Congress Association Andrzej Rychlewicz Zabidin Salleh Ákos Császár M. CALDAS National Academy of Sciences, India Indian Institute of Science, Bangalore*

this monograph is the first and an initial introduction to the theory of bitopological spaces and its applications in particular different families of subsets of bitopological spaces are introduced and various relations between

two topologies are analyzed on one and the same set the theory of dimension of bitopological spaces and the theory of baire bitopological spaces are constructed and various classes of mappings of bitopological spaces are studied the previously known results as well the results obtained in this monograph are applied in analysis potential theory general topology and theory of ordered topological spaces moreover a high level of modern knowledge of bitopological spaces theory has made it possible to introduce and study algebra of new type the corresponding representation of which brings one to the special class of bitopological spaces it is beyond any doubt that in the nearest future the areas of essential applications will be the theories of linear topological spaces and topological groups algebraic and differential topologies the homotopy theory not to mention other fundamental areas of modern mathematics such as geometry mathematical logic the probability theory and many other areas including those of applied nature key features first monograph is generalized lattices the first introduction to the theory of bitopological spaces and its applications

in this paper we introduce the notion of neutrosophic  $b$  open set pairwise neutrosophic  $b$  open set in neutrosophic bitopological spaces we have investigated some of their basic properties and established relation between the other existing notions

an asymmetric norm is a positive definite sublinear functional  $p$  on a real vector space  $x$  the topology generated by the asymmetric norm  $p$  is translation invariant so that the addition is continuous but the asymmetry of the norm implies that the multiplication by scalars is continuous only when restricted to non negative entries in the first argument the asymmetric dual of  $x$  meaning the set of all real valued upper semi continuous linear functionals on  $x$  is merely a convex cone in the vector space of all linear functionals on  $x$  in spite of these differences many results from classical functional analysis have their counterparts in the asymmetric case by taking care of the interplay between the asymmetric norm  $p$  and its conjugate among the positive results one can mention hahn banach type theorems and separation results for convex sets krein milman type theorems analogs of the fundamental principles open mapping closed graph and uniform boundedness theorems an analog of the schauder  $s$  theorem on the compactness of the conjugate mapping applications are given to best approximation problems and as relevant examples one

considers normed lattices equipped with asymmetric norms and spaces of semi lipschitz functions on quasi metric spaces since the basic topological tools come from quasi metric spaces and quasi uniform spaces the first chapter of the book contains a detailed presentation of some basic results from the theory of these spaces the focus is on results which are most used in functional analysis completeness compactness and baire category which drastically differ from those in metric or uniform spaces the book is fairly self contained the prerequisites being the acquaintance with the basic results in topology and functional analysis so it may be used for an introduction to the subject since new results in the focus of current research are also included researchers in the area can use it as a reference text

in this paper we built bitopological space on the concept of neutrosophic soft set we defined the basic topological concepts of this spaces which are  $n_3$  bi open set  $n_3$  bi closed set bi neutrosophic soft interior bi neutrosophic soft closure bi neutrosophic soft boundary bi neutrosophic soft exterior and we introduced their properties in addition we investigated the relations of these basic topological concepts with their counterparts in neutrosophic soft topological spaces and we introduced many examples

neutrosophic sets and systems has been created for publications on advanced studies in neutrosophy neutrosophic set neutrosophic logic neutrosophic probability neutrosophic statistics that started in 1995 and their applications in any field such as the neutrosophic structures developed in algebra geometry topology etc

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kelly introduced the concept of a bitopological space i e a triple  $X, \tau_1, \tau_2$  where  $X$  is a set and  $\tau_1, \tau_2$  are topologies on  $X$  he defined pairwise hausdorff pairwise regular pairwise normal spaces and obtained generalizations of several standard results such as urysohn s lemma tietze s extension theorem urysohn s metrization

theorem and the baire category theorem fletcher and lane independently defined pairwise completely regular and pairwise uniform spaces and proved their equivalence this thesis began in an attempt to define the concept of pairwise compactness in a bitopological space in a non trivial way after recalling known definitions and results in chapter 1 this is done in chapter 2 it is shown that the definition used here satisfies most of the requirements furthermore maximal and minimal bitopological spaces are investigated and the results are used in the sequel the results are then applied in chapter 3 to function space topologies which are studied in detail these function spaces are studied not only for pairwise continuous functions but also for certain types of non continuous functions such as pairwise connected and pairwise almost continuous functions analogously in bitopological space connected open topology and graph topology are considered in chapter 4 5 6 finally in the last chapter a new function space is introduced which is especially useful for the space of all functions which have at worst discontinuity of the first kind this sheds more light on the skorokhod convergence

the purpose of the present work is to introduce and investigate a new class of sets called  $i, j$  ps open sets and use this class to define and study new concepts in bitopological spaces such as continuity and separation axioms at the beginning of this work we define the class of  $i, j$  ps open sets which contained in the class of  $j$  preopen sets and also contained in the class of  $i, j$  gp open sets it is shown that the family of  $i, j$  ps open sets form a supratopology on  $x$  we prove that the family of  $i, j$  ps open sets and the family of  $j$  preopen sets are identical when  $x$  is a semi  $t_1$  space finally some separation axioms such as  $t_0$   $t_1$  and  $t_2$  spaces are defined in bitopological spaces also  $r_0$   $r_1$  and urysohn spaces are defined and the relation between them are found by using the new type of graph functions called  $i, j$  ps closed graph it is noticed that if  $x$  is  $i, j$  ps  $t_k$  then it is  $i, j$  ps  $t_{k-1}$  for  $k \geq 1$  it is proved that a bitopological space  $x$  is  $i, j$  ps  $t_1$  if the  $i, j$  ps derived set of every point of  $x$  is empty

this research article lays the foundation to propose the new concept of neutrosophic soft cubic topology here we focus on the systematic study of neutrosophic soft cubic sets and deduce various properties which are induced by them this enables us to introduce some equivalent characterizations and brings out the inter relations among them

the concept of ideals in topological spaces has been introduced and studied by

kuratowski 19 and vaidyanathasamy 24 hamlett and jankovi c see 12 13 17 and 18 used topological ideals to generalize many notions and properties in general topology

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