

# Commutative Algebra Exercises Solutions

Commutative Algebra Exercises Solutions Commutative Algebra Exercises Solutions This document provides solutions to a selection of exercises in commutative algebra. The exercises are chosen to cover a wide range of topics from basic definitions to more advanced concepts. The solutions are detailed and aim to provide a thorough understanding of the underlying concepts. This document is organized into sections each focusing on a specific topic in commutative algebra. Each section contains a brief overview of the topic followed by a selection of solved exercises.

**Sections**

- 1 Rings and Ideals**  
Overview: This section introduces the basic definitions of rings and ideals along with fundamental properties. Exercises: Prove that the set of even integers forms an ideal in the ring of integers. Determine whether the ideal generated by  $x^2 + 1$  is a prime ideal in the polynomial ring  $\mathbb{Z}[x]$ . Show that the intersection of two ideals is an ideal. Find all the ideals of the ring  $\mathbb{Z}/6\mathbb{Z}$ .
- 2 Modules and Homomorphisms**  
Overview: This section introduces the concept of modules, a generalization of vector spaces over fields. Homomorphisms between modules are also discussed. Exercises: Determine whether the set of all polynomials with even coefficients forms a submodule of the module of all polynomials over the ring of integers. Show that the set of all matrices of the form  $\begin{pmatrix} a & b & 0 \\ 0 & c & 0 \end{pmatrix}$  where  $a, b, c$  are real numbers forms a module over the ring of real numbers. Prove that the homomorphism theorem holds for modules. Determine the kernel and image of the homomorphism  $\phi: \mathbb{Z}/2\mathbb{Z} \rightarrow \mathbb{Z}$  defined by  $\phi(a) = a^2$ .
- 3 Localization and Quotient Rings**  
Overview: This section explores the concepts of localization and quotient rings which provide tools for constructing new rings from existing ones. Exercises: Find the localization of the ring of integers at the prime ideal  $2$ . Show that the quotient ring  $\mathbb{Z}_{(2)}$  is isomorphic to  $\mathbb{Z}/2\mathbb{Z}$ .
- 4 Noetherian Rings and Modules**  
Overview: This section introduces the concept of Noetherian rings and modules which are crucial for studying ideals and submodules. Exercises: Show that the ring of integers is a Noetherian ring. Prove that any finitely generated module over a Noetherian ring is Noetherian. Demonstrate that the Hilbert Basis Theorem holds for polynomial rings over Noetherian rings. Find a ring that is not Noetherian.
- 5 Integral Domains and Field Extensions**  
Overview: This section delves into integral domains and field extensions which are central to algebraic number theory and algebraic geometry. Exercises: Show that the ring of Gaussian integers is an integral domain. Determine the degree of the field extension  $\mathbb{Q}(\sqrt{2})/\mathbb{Q}$ . Prove that any finite integral domain is a field. Find the minimal polynomial of  $\sqrt[3]{2}$  over the field of rational numbers.
- 6 Dimension Theory**  
Overview: This section introduces the notion of dimension for rings and modules which measures their complexity in terms of chains of ideals or submodules. Exercises: Calculate the Krull dimension of the ring of integers. Determine the dimension of the polynomial ring  $\mathbb{C}[x, y]$ . Prove that the dimension of a Noetherian ring is finite. Show that the dimension of a module is less than or equal to the dimension of the ring.
- 7 Primary Decomposition and Associated Primes**  
Overview: This section explores primary decomposition, a powerful tool for understanding ideals in commutative rings and the concept of associated primes. Exercises: Find a primary decomposition of the ideal  $x^2, xy$  in the polynomial ring  $\mathbb{C}[x, y]$ . Determine the associated primes of the ideal  $x^2, xy$  in  $\mathbb{C}[x, y]$ . Prove that the radical of a primary ideal is a prime ideal.
- 8 Unique Factorization Domains and Dedekind Domains**  
Overview: This section discusses the concept of unique factorization domains (UFDs) which generalize the familiar factorization of integers into primes and Dedekind domains, a class of rings with important properties in algebraic number theory.

Exercises Prove that the polynomial ring  $\mathbb{C}[x]$  is a UFD Show that the ring of integers of a quadratic field is a Dedekind domain Find the prime factorization of the ideal  $2$  in the ring of integers of the quadratic field  $\mathbb{Q}(\sqrt{5})$  Determine whether the ring  $\mathbb{Z}[\sqrt{3}]$  is a UFD 9 Modules over Principal Ideal Domains Overview This section focuses on modules over principal ideal domains (PIDs) which have a rich structure and play a significant role in linear algebra Exercises Classify all finitely generated modules over the ring of integers Prove that any finitely generated module over a PID is a direct sum of cyclic modules Determine the invariant factors of the module  $\mathbb{Z}^6 \oplus \mathbb{Z}^{12}$  Find the Smith normal form of the matrix  $\begin{pmatrix} 2 & 4 & 6 & 8 \end{pmatrix}$  over the ring of integers 10 Applications of Commutative Algebra Overview This section explores various applications of commutative algebra in other areas of mathematics including algebraic geometry number theory and coding theory 4 Exercises Use Hilbert's Nullstellensatz to show that the ideal  $x^2y^2 + 1$  in  $\mathbb{C}[x, y]$  defines the unit circle in the complex plane Apply the Chinese Remainder Theorem to solve the system of congruences  $x \equiv 1 \pmod{2}$ ,  $x \equiv 2 \pmod{3}$ ,  $x \equiv 3 \pmod{5}$  Use the theory of Gröbner bases to find the solution to the system of equations  $x^2 + y^2 = 1$ ,  $x + y = 1$  Note The selection of exercises and their level of difficulty will vary depending on the target audience and the scope of the course This document provides a general framework for structuring a collection of commutative algebra exercise solutions

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this book comprises well over three hundred exercises in matrix algebra and their solutions the

exercises are taken from my earlier book matrix algebra from a statistician's perspective they have been restated as necessary to make them comprehensible independently of their source to further insure that the restated exercises have this stand alone property i have included in the front matter a section on terminology and another on notation these sections provide definitions descriptions comments or explanatory material pertaining to certain terms and notational symbols and conventions from matrix algebra from a statistician's perspective that may be unfamiliar to a nonreader of that book or that may differ in generality or other respects from those to which he she is accustomed for example the section on terminology includes an entry for scalar and one for matrix these are standard terms but their use herein and in matrix algebra from a statistician's perspective is restricted to real numbers and to rectangular arrays of real numbers whereas in various other presentations a scalar may be a complex number or more generally a member of a field and a matrix may be a rectangular array of such entities

this book is mainly intended for first year university students who undertake a basic abstract algebra course as well as instructors it contains the basic notions of abstract algebra through solved exercises as well as a true or false section in each chapter each chapter also contains an essential background section which makes the book easier to use

over 300 unusual problems ranging from easy to difficult involving equations and inequalities diophantine equations number theory quadratic equations logarithms more detailed solutions as well as brief answers for all problems are provided

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problem solving approaches often find a path from the givens to the unknown or from the unknown to the givens this study explores those approaches using written protocols of navy subjects while solving for the numerical value of an unknown from several knowns and a system of equations for a subset of the items solutions require subjects to retain expressions with both literals and numerals successful and unsuccessful students were similar in their preference to work backward and to use literals rather than numerals and in the efficacy of their solutions students acquired successful solution methods during instruction and used them consistently author

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